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Further Results Based on Chernoff-type Inequalities

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Abstract

In this paper, we address questions dealing with characterizations based on Chernoff-type moment inequalities and their variants and establish, via the approach of Alharbi & Shanbhag (1996), a general theorem extending, among others, various results of Cacoullos & Papathanasiou (1995a,1995b).

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Keywords: The Chernoff-inequality, Variance Bounds, Lebesgue-Stieltjes Measure, Hazard measure, Characterizations, The Cox Representation.

1 Introduction

There is an extensive literature dealing with upper and lower bounds for the variance of a function of a random variable : Chernoff (1981) gave a bound for the variance of an absolutely continuous function (w.r.t. Lebesgue measure) of a normal random variable. Cacoullos (1982) and Klaassen (1985) obtained variations of the inequality relative to other distributions and also gave the corresponding lower bounds. Borovkov & Utev (1983), and several others, gave characterizations via Chernoff-type inequalities. During the last fifteen years or so, many papers have appeared on modified versions or variants of the Chernoff inequality and

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related characterizations. Variations or extended versions of these latter results and characterizations relative to Chernoff-type inequalities have been obtained by Cacoullos & Papathanasiou (1985,1989,1992,1995a,1995b), Chen (1982), Koicheva (1993), Parakasa Rao & Sreehari (1986,1987,1995), Srivastava & Sreehari (1987,1990), Parakasa Rao (1992), Purkayata & Bhandari (1990), Hwang & Sheu (1987) and Korwar (1991) among others. Alharbi & Shanbhag (1996) extended some of these results to a more general set-up relaxing the assumption that the distribution is absolutely continuous (w.r.t. Lebesgue measure) or it is purely discrete.

We now use the Alharbi-Shanbhag ideas to extend and unify further results in the literature, including among others, those of Cacoullos & Papathanasiou (1995a,1995b). We also give here a relevant representation concerning the distributions that are characterized. The results in Cacoullos & Papathanasiou (1995a,1995b), in turn, subsume many of the earlier characterizations based on variance bounds.

2 General Characterizations Based on the Chernoff-type Inequality

Alharbi & Shanbhag (1996) extended the theorems for distributions based on a version of the Chernoff inequality to the case where distributions are not necessarily purely discrete or absolutely continuous. Among the theorems they have established is the following theorem :

Theorem 2.1 *Let F^* be a non-constant Lebesgue-Stieltjes measure function on $\Re$ and ν_{F^*} be the measure on the Borel $\sigma-$ field of $\Re$ determined by it. Let X be an r.v. such that $E\{F^*(X)\} = \mu$, and $E(\{F^*(X)\}^2) < \infty$ and w be a Borel measurable function such that $w(X) > 0$ a.s. and $Var\{F^*(X)\} = E\{w(X)\}$. Further let τ be the class of real-valued absolutely continuous functions g with Radon-Nikodym derivative g' w.r.t. the measure ν_{F^*} (i.e. such that $g(b) - g(a) = \int_{(a,b]} g'(x) d\nu_{F^*}(x)$ for all a and b with $a < b$), satisfying $E(\{g(X)\}^2) < \infty$ and $0 < E\{w(X)[g'(X)]^2\} < \infty$. Then*

$$\sup_{g\in\tau} \frac{Var[g(X)]}{E\{w(X)[g'(X)]^2\}} = 1, \tag{1}$$

if and only if

$$w(x)dF(x) = \{\int_{[x,\infty)} [F^*(z) - \mu]dF(z)\} d\nu_{F^*}(x), \; x \in \Re, \tag{2}$$

where F is the df of the r.v. X .

They also established that if X, w, g and τ are as in the above theorem but with $E\{w(X) \mid g'(X) \mid\} < \infty$ and $E\{w(X)g'(X)\} \neq 0$ in place of $0 < E\{w(X)[g'(X)]^2\} < \infty$, then the assertion

of the theorem holds with

$$\inf_{g\in\tau}\frac{Var[g(X)]Var[F^*(X)]}{E^2\{w(X)g'(X)\}}=1, \tag{3}$$

in place of (1). (Incidentally, this latter result is our Corollary 2.6 given below.)

General theorems of Alharbi & Shanbhag (1996), subsume known results relative to distributions that are absolutely continuous w.r.t. Lebesgue measure and those relative to purely discrete distributions. Using the ideas in this latter paper, we now extend major results of Cacoullos & Papathanasiou (1995a,1995b) on covariance identities and variance bounds, to have results essentially in the spirit of the results of Alharbi & Shanbhag (1996) stated above. We also give a representation for distributions, linked with our results.

Theorem 2.2 *Let F^* be a non-constant Lebesgue-Stieltjes measure function on $\Re$ and ν_{F^*} be the measure on the Borel $\sigma-$ field of $\Re$ determined by it, and let h^* and Z be Borel measurable functions. Let X be an r.v. with df F such that $h^*(X)$ is integrable with $\mu^*=E[h^*(X)]$ and $E\{|Z(X)|I_{\{X\in(a,b)\}})<\infty$ for every $-\infty<a<b<\infty$ and satisfying the condition that $\liminf_{x\to\infty}(h^*(x)-\mu^*)>0$ if the right extremity of F equals ∞, and the condition that $\liminf_{x\to-\infty}(\mu^*-h^*(x))>0$ if the left extremity of F equals $-\infty$. Further let τ be the class of real-valued absolutely continuous functions g with Radon-Nikodym derivative g' w.r.t. the measure ν_{F^*} (i.e. such that $g(b)-g(a)=\int_{(a,b]}g'(x)d\nu_{F^*}(x)$ for all a and b with $a<b$). Then, we have the condition*

$$Cov\{g(X),\ h^*(X)\}=E\{Z(X)g'(X)\}, \tag{4}$$

met for all g with $E(\mid Z(X)g'(X)\mid)<\infty$, if and only if

$$Z(x)dF(x)=\{\int_{[x,\infty)}[h^*(z)-\mu^*]dF(z)\}d\nu_{F^*}(x),\ x\in\Re. \tag{5}$$

(We read (4) as the condition where the left hand side of the identity is well defined and equals the right hand side of the identity.)

Proof: The " if " part can be proved via an extended version of the argument as in Alharbi & Shanbhag (1996) by applying Fubini's theorem as follows:

(5) implies that

$$
\begin{aligned}
E\{Z(X)g'(X)\} &= \int_{\Re} g'(x)\{\int_{[x,\infty)}[h^*(z)-\mu^*]dF(z)\}d\nu_{F^*}(x) \\
&= \int_{[a,\infty)} g'(x)\{\int_{[x,\infty)}[h^*(z)-\mu^*]dF(z)\}d\nu_{F^*}(x) \\
&+ \int_{(-\infty,a)} g'(x)\{\int_{(-\infty,x)}[\mu^*-h^*(z)]dF(z)\}d\nu_{F^*}(x) \\
&= \int_{[a,\infty)} \big(g(z)-g(a)\big)[h^*(z)-\mu^*]dF(z) \\
&+ \int_{(-\infty,a)} \big(g(a)-g(z)\big)[\mu^*-h^*(z)]dF(z) \\
&= Cov\{h^*(X),g(X)\}. \qquad (6)
\end{aligned}
$$

It is easily seen that Fubini's theorem applies here. The " only if " part could be proved as follows :

We have

$$E\{Z(X)g'(X)\} = \int_{\Re} g'(x)Z(x)dF(x)$$

and with $a \in \Re$,

$$
\begin{aligned}
Cov\{g(X),h^*(X)\} &= E\{g(X)[h^*(X)-E\big(h^*(X)\big)]\} \\
&= \int_{\Re} g(x)[h^*(x)-\mu^*]dF(x) \\
&= \int_{\Re} \big(g(a)+\int_{(a,x]} g'(y)d\nu_{F^*}(y)\big)[h^*(x)-\mu^*]dF(x) \\
&= \int_{\Re}\int_{(a,x]} g'(y)d\nu_{F^*}(y)[h^*(x)-\mu^*]dF(x) \\
&= \int_{(a,\infty)}\int_{(a,x]} g'(y)d\nu_{F^*}(y)[h^*(x)-\mu^*]dF(x) \; nonumber \\
&- \int_{(-\infty,a]}\int_{(x,a]} g'(y)d\nu_{F^*}(y)[h^*(x)-\mu^*]dF(x) \\
&= \int_{(a,\infty)} \big(\int_{[y,\infty)}[h^*(x)-\mu^*]dF(x)\big)g'(y)d\nu_{F^*}(y) \\
&+ \int_{(-\infty,a]} \big(\int_{[y,\infty)}[h^*(x)-\mu^*]dF(x)\big)g'(y)d\nu_{F^*}(y) \\
&= \int_{\Re} \big(\int_{[y,\infty)}[h^*(x)-\mu^*]dF(x)\big)g'(y)d\nu_{F^*}(y).
\end{aligned}
$$

Let $-\infty < a < b < \infty$ and let g be absolutely continuous w.r.t. ν_{F^*} such that

$$g'(x) = \begin{cases} 0 & \text{if } x \notin (a,b) \\ 1 & \text{if } x \in (a,b). \end{cases}$$

Then

$$\int_{(a,b)} Z(x)dF(x) = \int_{(a,b)} \big(\int_{[x,\infty)}[h^*(y)-\mu^*]dF(y)\big)d\nu_{F^*}(x),$$

for all arbitrary $a,\ b > 0$. This implies that

$$\Big(\int_{[x,\infty)} \big(h^*(t) - \mu^*]\big) dF(t)\Big) d\nu_{F^*}(x) = Z(x)dF(x),$$

which is (5).

Theorem 2.3 *Let $X,\ g,\ \tau,\ Z$ and h^* be as defined in Theorem 2.2, but additionally with h^* absolutely continuous w.r.t. ν_{F^*}, $g(X)$ square integrable and $E\{Z(X)g'(X)\}$ defined and nonzero for every $g \in \tau$, and $h^*(X)$ nondegenerate integrable satisfying*

$$Var\{h^*(X)\} = E\big(Z(X)h^{*\prime}(X)\big). \tag{7}$$

Then

$$\inf_{g\in\tau} \frac{Var[g(X)]Var[h^*(X)]}{E^2\{Z(X)g'(X)\}} = 1, \tag{8}$$

if and only if (5) holds.

Proof: We shall first establish the " if " part; note that (8) is equivalent to

$$Var\{g(X)\}Var\{h^*(X)\} \geq E^2\{Z(X)g'(X)\},\ g \in \tau, \tag{9}$$

since the equality in (9) holds if $g = h^*$. Clearly, if we assume (5), we have

$$E\{Z(X)g'(X)\} = Cov[g(X), h^*(X)], \tag{10}$$

as seen in Theorem 2.2. Note now that the equality in (9) holds if $g(.) = h^*(.)$. Hence, under the stated assumptions,

$$\begin{aligned} E^2\{Z(X)g'(X)\} &= \{Cov[g(X), h^*(X)]\}^2 \\ &\leq Var[g(X)]Var[h^*(X)], \end{aligned} \tag{11}$$

with equality in (11) if $g = h^*$. This establishes the " if " part of the theorem. The " only if " part of the theorem may be proved by extending the method of Alharbi & Shanbhag (1996) as follows:

Let (a, b) be a bounded open interval and

$$k(x) = \begin{cases} 0 & \text{if } x \notin (a,b) \\ 1 & \text{if } x \in (a,b). \end{cases}$$

For any real θ, we define

$$g(x) = h^*(x) - \mu^* + \theta \int_{(-\infty,x]} k(y)d\nu_{F^*}(y), \quad x \in \Re.$$

Clearly, in view of the relations (7) and (8)

$$\begin{aligned}
& Var\{h^*(X)\} + \theta^2 Var\{\int_{(-\infty,X]} k(y)d\nu_{F^*}(y)\} \\
+ \; & 2\theta Cov\{h^*(X) - \mu^*, \int_{(-\infty,X]} k(y)d\nu_{F^*}(y)\} \\
\geq \; & E[Z(X)h^{*\prime}(X))] + \frac{1}{Var[h^*(X)]}\theta^2 E^2[Z(X)k(X)] \\
+ \; & 2\theta E[Z(X)k(X)]. \qquad (12)
\end{aligned}$$

We see that

$$\begin{aligned}
& \theta^2\big(Var\{\int_{(-\infty,X]} k(y)d\nu_{F^*}(y)\} - \frac{1}{Var[h^*(X)]}\{E^2[Z(X)k(X)]\}\big) \\
+ \; & 2\theta\big(Cov\{h^*(X) - \mu^*, \int_{(-\infty,X]} k(y)d\nu_{F^*}(y)\} \\
- \; & E[Z(X)k(X)]\big) \geq 0. \qquad (13)
\end{aligned}$$

Because (13) holds for all θ, it implies

$$Cov\{h^*(X) - \mu^*, \int_{(-\infty,X]} k(y)d\nu_{F^*}(y)\} = E[Z(X)k(X)].$$

In view of Fubini's theorem

$$\int_{(a,b)} Z(x)dF(x) = \int_{(a,b)} \{\int_{[x,\infty)} [h^*(z) - \mu^*]dF(z)\}d\nu_{F^*}(x),$$

which implies (5). Hence we have the theorem.

Corollary 2.4 *Let X and g and τ be as defined the same as Theorem 2.1, but with $E\{w(X) \mid g'(X) \mid\} < \infty$. Then*

$$Cov\{g(X),\ F^*(X)\} = E\{w(X)g'(X)\}, \qquad (14)$$

for all g if and only if (2) holds.

Proof: The corollary follows easily from Theorem 2.2 on taking $h^*(.) = F^*(.)$ and noting that the assumptions of the theorem are met.

Remark 2.5 One could also use an argument based on Fourier transforms to prove the " only if " part of Theorem 2.2 and hence implicity of Corollary 2.4.

Corollary 2.6 (Alharbi & Shanbhag (1996)). *Let X, h^*, Z, g and τ be as defined in Theorem 2.3, but with $h^* = F^*$. Then*

$$\inf_{g\in\tau} \frac{Var[g(X)]Var[F^*(X)]}{E^2\{Z(X)g'(X)\}} = 1, \qquad (15)$$

if and only if (2) with Z in place of w holds.

Proof: The corollary follows immediately from Theorem 2.3 on taking $h^* = F^*$.

The following theorem gives a criterion under which any F satisfying (4) is identified by (h^*, F^*, μ^*, Z) :

Theorem 2.7 *Suppose $a \in \Re$, and in (5), $Z(.) \neq 0$ a.e. ν_{F^*}, and $(Z(.))^{-1}$ and $\frac{h^*(.)-\mu^*}{Z(.)}$ are ν_{F^*}-integrable on every bounded interval, such that, for each $x \in \Re$,*

$$\int_{[a,x)} \int_{[a,y_n)} \cdots \int_{[a,y_2)} \Big(\prod_{i=1}^{n-1} \frac{h^*(y_i) - \mu^*}{Z(y_i)}\Big) \frac{Z(y_1)}{Z(y_n)} dF(y_1) d\nu_{F^*}(y_2) \ldots d\nu_{F^*}(y_n) \to 0 \text{ as } n \to \infty, \quad (16)$$

where we define $\int_{[a,y)} = - \int_{[y,a)}$ if $y < a$. Then, if F is a distribution function satisfying (5), we have for every $x \in \Re$,

$$\overline{F}(x) = \overline{F}(a) + C \sum_{n=1}^{\infty} (-1)^n \int_{[a,x)} \int_{[a,y_n)} \cdots \int_{[a,y_2)} \Big(\prod_{i=1}^{n-1} \frac{h^*(y_i) - \mu^*}{Z(y_i)}\Big) \frac{1}{Z(y_n)} d\nu_{F^*}(y_1) \ldots d\nu_{F^*}(y_n), \quad (17)$$

where $\int_{[a,y_n)} \cdots \int_{[a,y_2)} \big(\prod_{i=1}^{n-1} \frac{h^*(y_i)-\mu^*}{Z(y_i)}\big) \frac{1}{Z(y_n)}$ is to be read as $\frac{1}{Z(y_1)}$ if $n = 1$, $\overline{F}(x) = 1 - F(x-)$, $x \in \Re$, and $C = E\{(h^*(X) - \mu^*) I_{\{X \geq a\}}\}$.

Proof: We have, under the stated assumptions,

$$\begin{aligned}
\overline{F}(x) &= \int_{[x,\infty)} \frac{1}{Z(y_2)} \Big(\int_{[y_2,\infty)} (h^*(y_1) - \mu^*) dF(y_1) \Big) d\nu_{F^*}(y_2) \\
&= \overline{F}(a) - \int_{[a,x)} \frac{1}{Z(y_2)} \Big(\int_{[y_2,\infty)} (h^*(y_1) - \mu^*) dF(y_1) \Big) d\nu_{F^*}(y_2) \\
&= \overline{F}(a) - \int_{[a,x)} \frac{1}{Z(y_2)} \Big(C - \int_{[a,y_2)} (h^*(y_1) - \mu^*) dF(y_1) \Big) d\nu_{F^*}(y_2) \\
&= \overline{F}(a) - C \int_{[a,x)} \frac{1}{Z(y_1)} d\nu_{F^*}(y_1) + \int_{[a,x)} \int_{[a,y_2)} \frac{(h^*(y_1) - \mu^*)}{Z(y_2)} dF(y_1) d\nu_{F^*}(y_2) \\
&\vdots \\
&= \overline{F}(a) + C \sum_{n=1}^{k-1} (-1)^n \int_{[a,x)} \int_{[a,y_n)} \cdots \int_{[a,y_2)} \Big(\prod_{i=1}^{n-1} \frac{h^*(y_i) - \mu^*}{Z(y_i)}\Big) \frac{1}{Z(y_n)} d\nu_{F^*}(y_1) \ldots d\nu_{F^*}(y_n) \\
&+ (-1)^k \int_{[a,x)} \int_{[a,y_k)} \cdots \int_{[a,y_2)} \Big(\prod_{i=1}^{k-1} \frac{h^*(y_i) - \mu^*}{Z(y_i)}\Big) \frac{Z(y_1)}{Z(y_k)} dF(y_1) d\nu_{F^*}(y_2) \ldots d\nu_{F^*}(y_k) (\text{with } k \geq 2) \\
&\to \overline{F}(a) + C \sum_{n=1}^{\infty} (-1)^n \int_{[a,x)} \int_{[a,y_n)} \cdots \int_{[a,y_2)} \Big(\prod_{i=1}^{n-1} \frac{h^*(y_i) - \mu^*}{Z(y_i)}\Big) \frac{1}{Z(y_n)} d\nu_{F^*}(y_1) \ldots d\nu_{F^*}(y_n), \text{as } k \to \infty);
\end{aligned} \quad (18)$$

on using (16). Consequently, it follows that the assertion holds. (Note that (17) implies that $\overline{F}$ is determined by (h^*, F^*, μ^*, Z) because the fact that $\overline{F}(x) \to 0$ as $x \to \infty$ and $\overline{F}(x) \to 1$ as $x \to -\infty$ implies in view of (17) also that $\overline{F}(a)$ and c are determined by (h^*, F^*, μ^*, Z).)

3 Cacoullos & Papathanasiou (1995a, 1995b) Results as Corollaries

Cacoullos & Papathanasiou (1995a) generalized the covariance identity for univariate random variables and used them to obtain several characterizations. Here, lower and upper variance bounds were derived by them using a covariance identity, appearing in Cacoullos & Papthanasiou (1995b).

In this section, we establish that some of the results of the papers of Cacoullos & Papthanasiou (1995a,1995b) are corollaries to the results established in the last section. The following corollary is essentially due to Cacoullos & Papathanasiou (1995a) on taking into account what is observed in Remark 3.3.

Corollary 3.1 *Let h and Z be absolutely continuous Borel measurable functions (w.r.t. Lebesgue measure) and let X be an r.v. with df F such that $h(X)$ is integrable with $\mu = E[h(X)]$ and $E\{|Z(X)|I_{\{X\in(a,b)\}}) < \infty$ for every $-\infty < a < b < \infty$ and satisfying the condition that $\liminf_{x\to\infty}(h(x) - \mu) > 0$ if the right extremity of F equals ∞, and the condition that $\liminf_{x\to-\infty}(\mu - h(x)) > 0$ if the left extremity of F equals $-\infty$. Further let τ be the class of real-valued absolutely continuous functions g with Radon-Nikodym derivative g' w.r.t. Lebesgue measure. Then, we have the condition*

$$Cov\{g(X),\ h(X)\} = E\{Z(X)g'(X)\}, \tag{19}$$

met for all g with $E(\mid Z(X)g'(X)\mid) < \infty$, if and only if

$$Z(x)dF(x) = \{\int_{[x,\infty)}[h(z) - \mu]dF(z)\}dx,\ x \in \Re. \tag{20}$$

Corollary 3.2 *Let X, g, τ, Z, and h be as defined in Corollary 3.1, but additionally with $Z(X)g'(X)$ and $h^2(X)$ integrable, and $E\{Z(X)g'(X)\} \neq 0$ and $V[h(X)] = E[Z(X)h'(X)]$. Then*

$$Var[g(X)] \geq \frac{E^2[Z(X)g'(X)]}{E[Z(X)h'(X)]}, \tag{21}$$

if and only if

$$Z(x)dF(x) = \{\int_{[x,\infty)}[h(y) - E(h)]dF(y)\}dx. \tag{22}$$

Equality holds if and only if $g(.) = c_1h(.) + c_2$.

Proof: The result follows from Theorem 2.3 on taking $F^*(x) = x, \ x \in \Re$.

Remark 3.3 Corollaries 3.1 and 3.2, assuming a priori F to be absolutely continuous, were essentially obtained by Cacoullos & Papathanasiou (1995a, 1995b). Also, they had established the specialized versions of Theorems 2.2 and 2.3 when $F^*(x) = [x], \ x \in \Re$ (where [.] refers to the integer part). Incidentally the arguments used by Cacoullos & Papathanasiou (1995a, 1995b) to get their results are based on an implicit application of Fubini's theorem, and the authors in question seem to have not stated explicitly the conditions under which the theorem is applicable. In our results, we have put conditions so that the theorem works ; one could obviously choose different conditions for this purpose.

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